REPORT DOCUMENTATION PAGE

Form Approved
OMB NO. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE July 15, 2004	3. REPORT TYPE AND DATES COVERED Final Progress Report 9/1/00-6/30/04

4. TITLE AND SUBTITLE
NURBS-Wavelets for surface design, analysis, visualization, and applications

5. FUNDING NUMBERS
DAAD19-00-1-0512

6. AUTHOR(S)
Charles K. Chui

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
The Curators of the University of Missouri-St. Louis, One University Blvd., St. Louis, MO 63121-4401

8. PERFORMING ORGANIZATION REPORT NUMBER 4G315

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

U. S. Army Research Office
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. 20040810 059

40095.5-MA

11. SUPPLEMENTARY NOTES
The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

12 a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution unlimited.

12 b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

The central theme of our research program is "splines and wavelets," with applications in two specific areas: signal processing and computer graphics. New mathematical theories on affine and nonstationary wavelets under the tight frame structure have been developed. While our theory on affine wavelets is quite complete, including arbitrary matrix dilation, vanishing moment recovery, oversampling theory, and approximation duals; our theory of nonstationary wavelets is a pioneer work that opens up a new research area in computational mathematics. In particular, since our nonstationary consideration applies to all B-splines on arbitrary nested knot sequences, our wavelet theory extends the well-established theory of spline functions, and our new spline-wavelets with minimum support extend the current powerful spline tool box and are certainly compatible with the IGES and STEPS standards for the CAD/CAM industry. In applications to signal processing, we have also developed mathematical theories on localized cosines and Gabor frames for time-frequency localization as well as balanced multi-wavelets and the new notion of ARM-lets for processing scalar-valued signals without the need of pre-processing. In computer graphics, we have introduced a direct approach to treating extraordinary points in surface subdivisions, based on refinable bivariate splines with small supports.

14. SUBJECT TERMS
Mathematical theory and methods, wavelets, splines, nonstationary wavelets, tight frames, sibling frames, signal processing, image analysis, subdivision schemes

15. NUMBER OF PAGES
7

16. PRICE CODE

17. SECURITY CLASSIFICATION OR REPORT	18. SECURITY CLASSIFICATION ON THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	**UNCLASSIFIED**	**UNCLASSIFIED**	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev.2-89)
Prescribed by ANSI Std. 239-18
298-102

REPORT DOCUMENTATION PAGE (SF298)
(Continuation Sheet)

1. Research publications of P.I., Charles Chui, under the sponsorship of ARO Grant #DAAD 19- 00-1-0512

A. Papers in Peer-Reviewed Journals:

1. C.K. Chui and W. He, Construction of multivariate tight frames via Kronecker products, *Appl. Comp. Harmonic Anal.* 11 (2001), 305-312.
2. C.K. Chui, W. He, and J. Stöckler, Compactly supported tight and sibling frames with maximum vanishing moments, *Appl. Comp. Harmonic Anal.* 13 (2002), 224-262.
3. C.K. Chui and Q. Sun, Tight frame oversampling and its equivalence to shift-invariance of frame, *Proc. of Amer. Math. Soc.* 131 (2003), 1527-1538.
4. C.K. Chui, W. He, J. Stöckler, and Q. Sun, Compactly supported tight affine frames with integer dilation and maximum vanishing moments, *Adv. Comp. Math.* 18 (2003), 159-187.
5. K. Bittner and C.K. Chui, Spline modulation of sinusoids for signal representation, *J. Fourier Anal. & Appl.* **9** (2003), 597—622.
6. K. Bittner and C.K. Chui, Formulation of localized cosine bases that preserve polynomial modulated sinusoids, *J. Fourier Anal. & Appl.*, accepted.
7. C.K. Chui and J.A. Lian, Construction of orthonormal multi-wavelets with additional vanishing, *Adv. Comp. Math.*, accepted.
8. J.-A. Lian and C.K. Chui, Balanced Multi-wavelets with short filters, *IEEE – Signal Processing Letters* **11**(2) (2004), 75-78.
9. J.-A. Lian and C.K. Chui, Analysis-ready multi-wavelets (Armlets) for processing scalar-valued signals, *IEEE Signal Processing Letters* **11**(2) (2004), 205-208.
10. C.K. Chui and J.Z. Wang, Shannon wavelet approach to sub-band coding, *Int'l J. Wavelets, Multiresolution and Inform Proc.* **1** (2003), 233-242.
11. C.K. Chui and Q.T. Jiang, Balanced multi-wavelets in R^s, *Math. Comp.*, accepted.
12. C.K. Chui, W. He, and J. Stöckler, Nonstationary tight wavelet frames, I: Bounded intervals, *Appl. Comp. Harmonic Anal.*, accepted.
13. C.K. Chui and Q.T. Jiang, Surface subdivision schemes generated by refinable bivariate spline function vectors, *Appl. Comp. Harmonic Anal.* **15** (2003), 147-162.
14. C.K. Chui and Q.T. Jiang, Refinable bivariate C^2-splines for surface subdivisions and multi-level data representation, *Math. Comp.*, accepted.
15. R. Garnett, T. Huegerich, C.K. Chui, and W. He, A universal noise removal algorithm with an impulse detector, *IEEE Trans. Image Processing*, accepted.

B. Papers Submitted to Peer-reviewed Journals:

16. C.K. Chui, W. He, and J. Stöckler, Nonstationary tight wavelet frames, II: Unbounded intervals, *Appl. Comp. Harmonic Anal.*, under review.

C. Peer-reviewed Papers Appeared in Edited Volumes:

17. C.K. Chui and J.-A. Lian, Multilevel structure of NURBS and formulation of NURBlets, in *Wavelet Analysis - 20 years Development*, D.X. Zhou (ed.), World Scientific, Singapore, 2001, pp. 23-38.
18. K. Bittner and C.K. Chui, Gabor frames with arbitrary windows, in *Approximation Theory X*, C.K. Chui, L.L. Schumaker, J. Stöckler (eds.), Vanderbilt University Press, 2002, pp. 41-50.
19. C.K. Chui and J.-A. Lian, Nonstationary wavelets and refinement sequences of nonuniform B-splines, in *Approximation Theory X*, C.K. Chui, L.L. Schumaker, J. Stöckler (eds.), Vanderbilt University Press, 2002, pp. 207-230.
20. C.K. Chui, W. He, and J. Stöckler, Tight frames with maximum vanishing moments and minimum support, in *Approximation Theory X*, C.K. Chui, L.L. Schumaker, J. Stöckler (eds.), Vanderbilt University Press, 2002, pp. 187-206.
21. C.K. Chui and J. Stöckler, Recent development of spline wavelet frames with compact, in *Beyond Wavelets*, G.V. Welland (ed.),Studies in Comput. Math. Series Vol. **10**, Elsevier, Amsterdams, 2003, pp. 269-342.
22. C.K. Chui and Q.T. Jiang, Multivariate balanced vector-valued refinable functions, in *Modern Developments in Multivariate Approximation Theory*, W. Haussmann, K. Jetter, M. Reimer, and J. Stöckler (eds.), ISNM Vol. **145**, Birhhäuser Verlag, Basel, 2003, pp. 71-102.

Brief descriptions of results

Splines and wavelets constitute the central theme of the research program under the sponsorship of ARO Grant #DAAD 19-00-1-0512. In particular, new wavelets and bivariate splines are introduced to meet the need of various applications mainly in signal/image processing and computer graphics. For example, an innovative concept of "refinable" spline function vectors enables such applications as surface subdivisions, and compactly supported wavelets associated with B-splines on arbitrary nested knot sequences are formulated in the tight and sibling frame structure for scattered data analysis. The 22 papers completed over the duration of the funding period can be divided into the following three categories.

(1.1) Mathematical Foundations

The development of mathematical theory and methods that will have direct applications to the advancement of the engineering fields of signal processing, image analysis, and computer graphics, constitute the core of our mathematical research program over the duration of the funding period. The research findings are classified into the following four areas, for convenience.

Area 1: NURBS and nonstationary wavelets

The papers in this area are [17, 19, 2, 21, 12, 16].

Wavelets and wavelet algorithms corresponding to NURBS were introduced in [17, 19]. The Unitary Matrix Extension Principle was extended to allow for the construction of tight and sibling frames with full order of vanishing moments [2, 21], and the "time-domain" formulation of our extension was further extended to allow for the construction of tight frames of spline-wavelets with nested knot sequences on bounded and unbounded intervals in [21, 12, 16]. This new theory is quite deep, most general, and complete with construction algorithms and pseudo-codes. Matrix calculus is used in [12] to replace the Fourier analysis in [2] for the bound interval setting. The notion of "Approximate Duals" corresponding to banded matrices is introduced to replace the inverse Gramian matrices so as to achieve local support and to preserve the polynomial moments. To extend to the study of tight frames of spline-wavelets on the unbounded intervals, a unified approach using kernel operators is introduced in [16]. This approach not only corresponds to the Fourier approach via the

Parsevel identity when the knots are equally spaced and extend from $-\infty$ to $+\infty$, it also generalizes the matrix calculus used in our study [12] for splines/wavelets on a bounded interval. It is interesting to observe, however, that in contrast to the study of orthogonal and bi-orthogonal wavelets on a bounded interval, it is not clear how to use the wavelets for the unbounded interval as interior wavelets for the bounded interval while constructing boundary wavelets to keep the tight frame structure. On the other hand, while wavelets for the bounded interval can be used as a starting point to study wavelets on the one-sided infinite intervals, this approach fails for the bi-infinite interval. In other words, our two papers [12] and [16] are independent studies of non-stationary wavelet tight frames, though they are presented as companions to each other. This is the most time consuming effort that has occupied the full funding period. The two papers [12] and [16] together are over 13 manuscript pages.

Area 2: Affine wavelets

The papers in this area are [1, 2, 20, 3, 4].

This is the most complete area of our research findings, including construction of multivariate multi-wavelets in [1], full characterizations of multivariate tight affine frames with matrix dilations [4], an innovative notion of vanishing moments recovery (VMR) functions for constructing affine frames with any desirable order of vanishing moments in [2], the study of such wavelets with minimum supports in [20], function and operator theory approaches including shift-invariant frame operators and oversampling in [3], and development of a stability theory when applied to wavelet decomposition of functions in Sobolev spaces in our most recent elaborate work. Our comprehensive paper [2] represents a parallel development by another elaborate paper of Daubechies et al., and constitutes the foundation of our more current development [12, 16] of non-stationary wavelet frames.

Area 3: Localized cosines and Gabor frames

The papers in this area are [18, 5, 6].

A fundamental theory concerning uniform bounds of time-frequency window sizes was developed in [18]. Spline-modulations with proper scaling are shown to provide asymptotically optimal frame bounds [5], and most recently, a new class of window functions was introduced for recovering modulated sinusoids of optimal order in [6]. This provides a transition between classical Fourier series and the recent development of the theory of localized cosines.

Area 4: Multi-wavelets

The papers in this area are [22, 7, 11].

Though the recently introduced concept of balanced multi-wavelets has made some significant impact on the application of vector-valued wavelets to processing scalar-valued data sets without the need of data preprocessing, such balanced wavelets have been very difficult to construct, since they are defined in the continuous-time domain. In fact, less than five examples are known in the open literature, and they are restricted to 1-balanced and 2-balanced two-dimensional vectors. A complete characterization is given in our work [11] to facilitate the constructive procedures as well as to extend to the multivariate setting with arbitrarily desirable centers of balancing. Bivariate spline examples are given [22], and an algorithm for generating vanishing moments of higher order with increasing vector dimension was developed in [7].

(1.2) Signal and Image Processing

The papers in this area are [10, 8, 9, 15].

In the development of the wavelet signal processing theory, a wavelet-packet approach was introduced in our work [10] to provide optimal sub-band coding bit-rates along with a Shannon wavelet library.

In the development of wavelet signal processing mathematical tools, the Selesnick construction of GHM-like orthogonal scaling function vectors was extended in our paper [8] by providing the corresponding balanced multi-wavelets along with matrix-valued filters. The fundamental tools for the study of this problem area were also presented in [8]. Since the concept of balancing can be realized only by nonlinear methods, a new concept, called Armlets (which stands for analysis-ready wavelets) was introduced in our paper [9] to allow for the development of linear methods, replacing the balancing approach for the first level of multi-wavelet decomposition, and allowing for a combination of high-order armlets and lower-order balanced multi-wavelet construction in general. This facilitates the procedure of constructing multi-wavelets for processing scalar-valued data.

For image processing, the so-called bilateral filter, which depends on both spatial domain smoothing and range enhancement of image edges and details, has recently been demonstrated to be even more effective than the well-known anisotropic diffusion approach. In our work [15], we introduced another statistical component to identify impulse noise pixels. With this extra component, we introduced a trilateral filter that can be used effectively to remove a mixture of white and impulse noises from images, while keeping the image details, particularly image edges, intact. This is a project the P.I. designed to train undergraduate student research. The paper [15], accepted for publication by *IEEE Image Processing*, is a joint work with two undergraduate students.

(1.3) Computer-Aided Surface Design

The papers in this area are [13, 14].

This exciting area of research only started in late 2002. It consolidates the P.I.'s expertise in "multivariate splines" and "wavelets" in introducing an innovative direct approach to constructing "coefficient stencils" of subdivision schemes for designing surfaces in 3-D of arbitrary topology and with any desirable order of smoothness. In the current literature, only indirect approaches are available. For example, for designing C^1-surfaces with extra-ordinary points (with valences different from 6 for triangular planar surfaces), the coefficient stencils require some change of weighted averages near extra-ordinary points in order to achieve C^1 smoothness. Furthermore, to be able to find weights for achieving C^2 smoothness is a "Holy Grail" problem in this field.

On the other hand, the direct approach we introduced is not restricted to the order of smoothness and it applies to extraordinary points of all valences. The key idea is to extend the cascade algorithm approach to matrix-valued multiresolution, with non-stationary scaling function vectors in the form of bivariate vertex splines (a notion introduced by the P.I. some 20 years ago). The refinable vertex splines can take on any desirable order of smoothness, with an appropriate polynomial degree, and they are constructed on some "almost regular" meshes that center at the extra-ordinary points. The preliminary mathematics are developed in [13], with the C^1 result for arbitrary topology presented in our continuing effort. The corresponding preliminary mathematics for the C^2 setting was developed in our paper [14]. However, although we have examples to show that this new approach is very promising, much greater effort is required in the future.

2. Scientific personnel supported by this project

C.K. Chui, P.I.
Tim Huegerich, undergraduate student
Roman Garnett, undergraduate student

3. Scientific activities

A. Invited Addresses

1. Plenary Speaker, Trends in Approximation Theory, Vanderbilt University, Nashville, Tennessee, May 17—20, 2000.
2. Plenary Speaker, International Conference on Multivariate Approximation, Haus Bommerholz, Germany, Sept. 22—27, 2002.
3. Principal Speaker, Grambrinus Conference on Wavelets and Mathematics of Imaging, University of Dortmund, Germany, April 24—25, 2003.
4. Invited Speaker, BIRS Conference on Applicable Harmonic Analysis, Banff, Alberta, Canada, June 8—12, 2003.
5. Invited Participant and Chair of Graphics Section, Geometry and Computer Graphics Workshop, Erbach, Germany, Sept. 28—Oct.3, 2003.
6. Plenary Speaker, International Conference on Numerical Methods in Imaging Science and Information Processing, Singapore, Dec. 15—19, 2003.
7. Minisymposium Speaker, Computer Aided Geometric Design Conference, SIAM Meeting, Seattle, Washington, Nov. 2—7, 2003.
8. Frontiers Lecture series Speaker (3 lectures), Texas A&M University, College Station, Texas, April 5—9, 2004.
9. Principle Speaker, University Library Lecture Series, Prairie View A&M University, Texas, April 6, 2004.
10. Distinguished University Lecture Speaker, Texas A&M International University, Laredo, Texas, April 15—16, 2004.
11. Plenary Speaker, NSF Show-Me Conference, Columbia, Missouri, June 3—5, 2004.
12. Plenary Speaker, International Congress of Chinese Mathematicians, Hong Kong, Dec. 17—22, 2004.

B. Organizer of International Conferences and Short Courses:

1. Tenth International Conference on Approximation Theory, with L.L. Schumaker and J. Stöckler, St. Louis, Missouri, March 26—29, 2001.
2. First International Conference on Computational Harmonic Analysis, with D.X. Zhou, Hong Kong, June 4—8, 2001.
3. Short Course Organizer on Refinable Bivariate Splines for Surface Subdivision, SIAM Conference on Geometric Design and Computing, Seattle, Washington, Nov. 10—23, 2003.
4. Eleventh International Conference on Approximation Theory, with M. Neamtu and L.L. Schumaker, Gatlinburg, Tennessee, May 18—22, 2004.
5. Second International Conference on Computational Harmonic Analysis, with A. Aldroubi, Nashville, Tennessee, May 24—30, 2004.
6. International Conference on Multivariate Approximation and Interpolation, with Applications, to take place at University of Hohenheim, Germany, October 13—17, 2004.

4. Honors/Awards

(1) Curators' Professorship awarded in January, 2003, by the University of Missouri System.
(2) Gambrinus Fellow/Prize, awarded in April 2003, by the University of Dortmund, Germany.